18. Predict the solution of $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = x^2$, when $y(0) = 0$ and $y'(0) = \frac{1}{2}$.

19. Demonstrate the orthogonality of Hermite polynomial.

20. Elaborate Green's function for three dimensional Helmhotz equation.

NOVEMBER/DECEMBER 2023

DPH11/GPH11 — MATHEMATICAL PHYSICS-I

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. List out the examples for linear vector spaces.

2. Infer the condition for vector field to be irrotational.

3. What are contravariant?

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4. Define Quotient law.

5. Solve the differential equation, $\frac{dy}{dx} = \frac{2x}{3y^2}$.

6. What is meant by "Order" of a differential equation.

7. Predict $P_2(x)$ using Rodrigue's formula for Legendre polynomial.

8. Recall the Rodrigue's formula for Hermite polynomial.

9. Show that $x\,\delta(x) = 0$, where $\delta(x)$ is Dirac delta function.

10. Solve $\int_{-\infty}^{\infty} x\,\delta(x-4)\,dx$.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Build the characteristic equation and find eigenvalues of $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.

Or

(b) Show that (0,1,1), (0,1,1) and (1,0,1) represent linearly independent vectors. Also check whether they are orthogonal or not.

12. (a) Compare covariant and contravariant tensors.

Or

(b) If A^{μ} and B_{μ} are any two vectors in which one is contravariant and other is covariant then prove that $A^{\mu}\,B_{\mu}$ is invariant.

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13. (a) Solve: $y'' + 25y = 0$.

Or

(b) Interpret the features of power series solution with one example.

14. (a) Inspect the orthogonality of Spherical Bessel functions.

Or

(b) Compile the generating function of Legendre polynomial.

15. (a) Dissect the solution of Strum-Liouville equation.

Or

(b) Outline the general proof of symmetry property of Green's function.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Formulate Cayley – Hamilton theorem along with necessary proof.

17. Explain symmetric and antisymmetric tensors.